Reference: DFE-00180-2013

Department for Education

Y6 Mathematics programme of study: matched with Key Objectives and Calculation links

National curriculum in England

Purpose of study

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Upper key stage 2 – years 5 and 6

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems. Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

Pupils should read, spell and pronounce mathematical vocabulary correctly.

Year 6 programme of study

Number – number and place value

Statutory requirements

Pupils should be taught to:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- **KO: round any whole number to a required degree of accuracy**
- **KO: use negative numbers in context, and calculate intervals across zero**
- solve number and practical problems that involve all of the above.

Notes and guidance (non-statutory)

Pupils use the whole number system, including saying, reading and writing numbers accurately.

Number – addition, subtraction, multiplication and division

Statutory requirements

Pupils should be taught to:

- **KO: multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication**
- **KO: divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context**
- **KO: divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context**
- **[KO: Continue to use all multiplication tables to 12 x 12 to maintain their fluency]**
- **[KO: Continue to practice formal methods of column addition, column subtraction, short multiplication and short division]**
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations

Statutory requirements

- **KO: Use the Y6 Mastery Map to solve problems involving addition, subtraction, multiplication and division**
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)

Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see Mathematics Appendix 1).

They undertake mental calculations with increasingly large numbers and more complex calculations.

Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.

Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.

Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.

Common factors can be related to finding equivalent fractions.

Number – fractions (including decimals and percentages)

Statutory requirements

Pupils should be taught to:

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]
- divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$]
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places

Statutory requirements

- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- **KO: recall and use equivalences between simple fractions, decimals and percentages, including in different contexts**

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Notes and guidance (non-statutory)

Pupils should practise, use and understand the addition and subtraction of fractions with different denominators by identifying equivalent fractions with the same denominator. They should start with fractions where the denominator of one fraction is a multiple of the other (for example, $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$) and progress to varied and increasingly complex problems.

Pupils should use a variety of images to support their understanding of multiplication with fractions. This follows earlier work about fractions as operators (fractions of), as numbers, and as equal parts of objects, for example as parts of a rectangle.

Pupils use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity (for example, if $\frac{1}{4}$ of a length is 36cm, then the whole length is 36 × 4 = 144cm).

They practise calculations with simple fractions and decimal fraction equivalents to aid fluency, including listing equivalent fractions to identify fractions with common denominators.

Pupils can explore and make conjectures about converting a simple fraction to a decimal fraction (for example, 3 ÷ 8 = 0.375). For simple fractions with recurring decimal equivalents, pupils learn about rounding the decimal to three decimal places, or other appropriate approximations depending on the context. Pupils multiply and divide numbers with up to two decimal places by one-digit and two-digit whole numbers. Pupils multiply decimals by whole numbers, starting with the simplest cases, such as 0.4 × 2 = 0.8, and in practical contexts, such as measures and money.

Pupils are introduced to the division of decimal numbers by one-digit whole number, initially, in practical contexts involving measures and money. They recognise division calculations as the inverse of multiplication.

Pupils also develop their skills of rounding and estimating as a means of predicting and checking the order of magnitude of their answers to decimal calculations. This includes rounding answers to a specified degree of accuracy and checking the reasonableness of their answers.

Ratio and proportion

Statutory requirements

Pupils should be taught to:

- **KO: solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts**
- **KO: solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison**
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

Notes and guidance (non-statutory)

Pupils recognise proportionality in contexts when the relations between quantities are in the same ratio (for example, similar shapes and recipes).

Pupils link percentages or 360° to calculating angles of pie charts.

Pupils should consolidate their understanding of ratio when comparing quantities, sizes and scale drawings by solving a variety of problems. They might use the notation *a*:*b* to record their work.

Pupils solve problems involving unequal quantities, for example, 'for every egg you need three spoonfuls of flour', '$\frac{3}{5}$ of the class are boys'. These problems are the foundation for later formal approaches to ratio and proportion.

Algebra

Statutory requirements

Pupils should be taught to:

- use simple formulae
- generate and describe linear number sequences
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns
- enumerate possibilities of combinations of two variables.

Notes and guidance (non-statutory)

Pupils should be introduced to the use of symbols and letters to represent variables and unknowns in mathematical situations that they already understand, such as:

- missing numbers, lengths, coordinates and angles
- formulae in mathematics and science
- equivalent expressions (for example, $a + b = b + a$)
- generalisations of number patterns
- number puzzles (for example, what two numbers can add up to).

Measurement

Statutory requirements

Pupils should be taught to:

- solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate
- **KO: use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places**
- convert between miles and kilometres
- recognise that shapes with the same areas can have different perimeters and vice versa
- recognise when it is possible to use formulae for area and volume of shapes
- calculate the area of parallelograms and triangles
- calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3].

Notes and guidance (non-statutory)

Pupils connect conversion (for example, from kilometres to miles) to a graphical representation as preparation for understanding linear/proportional graphs.

They know approximate conversions and are able to tell if an answer is sensible.

Using the number line, pupils use, add and subtract positive and negative integers for measures such as temperature.

Notes and guidance (non-statutory)

They relate the area of rectangles to parallelograms and triangles, for example, by dissection, and calculate their areas, understanding and using the formulae (in words or symbols) to do this.

Pupils could be introduced to compound units for speed, such as miles per hour, and apply their knowledge in science or other subjects as appropriate.

Geometry – properties of shapes

Statutory requirements

Pupils should be taught to:

- **KO: draw 2-D shapes using given dimensions and angles**
- recognise, describe and build simple 3-D shapes, including making nets
- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
- recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.

Notes and guidance (non-statutory)

Pupils draw shapes and nets accurately, using measuring tools and conventional markings and labels for lines and angles.

Pupils describe the properties of shapes and explain how unknown angles and lengths can be derived from known measurements.

These relationships might be expressed algebraically for example, $d = 2 \times r$; $a = 180 - (b + c)$.

Geometry – position and direction

Statutory requirements

Pupils should be taught to:

- describe positions on the full coordinate grid (all four quadrants)
- draw and translate simple shapes on the coordinate plane, and reflect them in the axes.

Notes and guidance (non-statutory)

Pupils draw and label a pair of axes in all four quadrants with equal scaling. This extends their knowledge of one quadrant to all four quadrants, including the use of negative numbers.

Pupils draw and label rectangles (including squares), parallelograms and rhombuses, specified by coordinates in the four quadrants, predicting missing coordinates using the properties of shapes. These might be expressed algebraically for example, translating vertex (a, b) to (a – 2, b + 3); (a, b) and (a + d, b + d) being opposite vertices of a square of side d.

Statistics

Statutory requirements

Pupils should be taught to:

- interpret and construct pie charts and line graphs and use these to solve problems
- calculate and interpret the mean as an average.

Notes and guidance (non-statutory)

Pupils connect their work on angles, fractions and percentages to the interpretation of pie charts.

Pupils both encounter and draw graphs relating two variables, arising from their own enquiry and in other subjects.

They should connect conversion from kilometres to miles in measurement to its graphical representation.

Pupils know when it is appropriate to find the mean of a data set.

Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division

This appendix sets out some examples of formal written methods for all four operations to illustrate the range of methods that could be taught. It is not intended to be an exhaustive list, nor is it intended to show progression in formal written methods. For example, the exact position of intermediate calculations (superscript and subscript digits) will vary depending on the method and format used.

For multiplication, some pupils may include an addition symbol when adding partial products. For division, some pupils may include a subtraction symbol when subtracting multiples of the divisor.

Addition and subtraction

789 + 642 becomes

$$\begin{array}{rrrr} & 7 & 8 & 9 \\ + & 6 & 4 & 2 \\ \hline 1 & 4 & 3 & 1 \\ & {\scriptstyle 1} & {\scriptstyle 1} & \end{array}$$

Answer: 1431

874 – 523 becomes

$$\begin{array}{rrrr} & 8 & 7 & 4 \\ - & 5 & 2 & 3 \\ \hline & 3 & 5 & 1 \end{array}$$

Answer: 351

932 – 457 becomes

$$\begin{array}{rrrr} & {\scriptstyle 8} & {\scriptstyle 12} & {\scriptstyle 1} \\ & \cancel{9} & \cancel{3} & 2 \\ - & 4 & 5 & 7 \\ \hline & 4 & 7 & 5 \end{array}$$

Answer: 475

932 – 457 becomes

$$\begin{array}{rrrr} & {\scriptstyle 1} & {\scriptstyle 1} & \\ & 9 & 3 & 2 \\ - & \cancel{4} & \cancel{5} & 7 \\ & & {\scriptstyle 5} & {\scriptstyle 6} \\ \hline & 4 & 7 & 5 \end{array}$$

Answer: 475

Short multiplication

24 × 6 becomes

$$\begin{array}{rrr} & 2 & 4 \\ \times & & 6 \\ \hline 1 & 4 & 4 \\ & {\scriptstyle 2} & \end{array}$$

Answer: 144

342 × 7 becomes

$$\begin{array}{rrrr} & 3 & 4 & 2 \\ \times & & & 7 \\ \hline 2 & 3 & 9 & 4 \\ & {\scriptstyle 2} & {\scriptstyle 1} & \end{array}$$

Answer: 2394

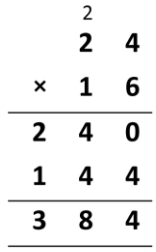
2741 × 6 becomes

$$\begin{array}{rrrrr} & 2 & 7 & 4 & 1 \\ \times & & & & 6 \\ \hline 1 & 6 & 4 & 4 & 6 \\ & {\scriptstyle 4} & {\scriptstyle 2} & & \end{array}$$

Answer: 16 446

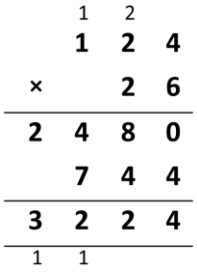
Long multiplication

24 × 16 becomes



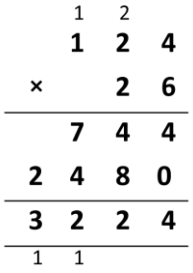
Answer: 384

124 × 26 becomes



Answer: 3224

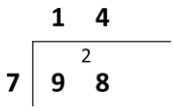
124 × 26 becomes



Answer: 3224

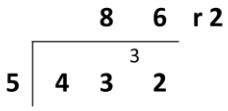
Short division

98 ÷ 7 becomes



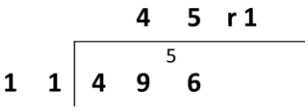
Answer: 14

432 ÷ 5 becomes



Answer: 86 remainder 2

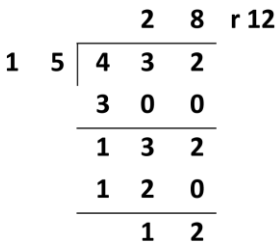
496 ÷ 11 becomes



Answer: $45\frac{1}{11}$

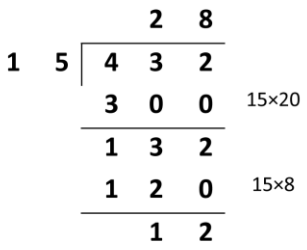
Long division

432 ÷ 15 becomes



Answer: 28 remainder 12

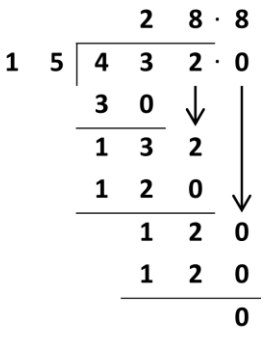
432 ÷ 15 becomes



$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes



Answer: 28·8

Reference: DFE-00180-2013